PREVIOUS YEAR QUESTION BANK

EXADEMY

Mathematics Optional Free Courses for UPSC and all state PCS

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ORDINARY DIFFERENTIAL EQUATIONS (ODE)

Q1. By eliminating the constants a, b obtain the differential equation for which $xy = ae^{x} + be^{-x} + x^{2}$ is a solution.

(Year 1992)

(20 Marks)

Q2. Find the orthogonal trajectory of the family of semi-cubical parabolas $ay^{2} = x^{2}, where\ a$ is a variable parameter.

(Year 1992)

(20 Marks)

Q3. Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents hyperbolas having the following lines as asymptotes $x + y = 0, 2x + y + 1 = 0$.

(Year 1992)

(20 Marks)

Q4. Solve the following differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$.

(Year 1992)

(20 Marks)

Q5. Find the curves for which the portion of *y-axis* cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact.

(Year 1992)

(20 Marks)

Q6. Solve the following differential equation: $(D^2 + 4)y = sin2x$, given that when $x=0$ then $y=0$ and $\frac{dy}{dx} = 2$.

(Year 1992)

(20 Marks)

Q7. Solve: $(D^2 - 1)y = xe^x + cos^2x$

(Year 1992)

(20 Marks)

Q8. Solve: $(x^2D^2 + xD - 4)y = x^2$

(Year 1992)

(20 Marks)

Q9. Determine the curvature for which the radius of curvature is proportional to the slope of the tangent.

(Year 1993)

(20 Marks)

Q10. Show that the system of co focal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal.

(Year 1993)

(20 Marks)

Q11. Solve $\left\{y\left(1 + \frac{1}{x}\right) + cosy\right\} dx + (x + logs - xsiny)dy = 0.$

(Year 1993)

(20 Marks)

Q12. Solve $y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 = y^2$.

(Year 1993)

(20 Marks)

Q13. Solve $\frac{d^2y}{dx^2} + {\omega_0}^2 y = acos\omega t$ and discuss the nature of solution as $\omega \xrightarrow{dt^2} \omega_0$.

(Year 1993)

(20 Marks)

Q14. Solve $(D^4 + D^2 + 1)y = e^{-x/2} \cos\left(\frac{x\sqrt{3}}{2}\right)$.

(Year 1993)

(20 Marks)

Q15. Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$

(Year 1994)

(20 Marks)

Q16. Show that if $\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$ is a function of *x* only, say, $f(x), then\ F(x) = e^{\int f(x)dx}$

Is an integration factor of $Pdx + Qdy = 0$.

(Year 1994)

(20 Marks)

Q17. Find the family of curves whose tangent from an angel $\frac{\pi}{4}$ with the hyperbola *xy=c*.

(Year 1994)

(20 Marks)

Q18. Transform the differential equation $\frac{d^2y}{dx^2}cosx + \frac{dy}{dx}sinx - 2ycos^3x = 2cos^5x$ into one having z as an independent variable where z=$sinx$ and solve it.

(Year 1994)

(20 Marks)

Q19. If $\frac{d^2x}{dt^2} + \frac{g}{b}(x - a) = 0$ (a, b and g being positive constants) and $x = a'$ and

$\frac{dx}{dt} = 0$ when t=0, show that $x = a + (a' - a)cost\sqrt{\frac{g}{b}}t.$

(Year 1994)

(20 Marks)

Q20. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}sin2x$ where $D \equiv \frac{dy}{dx}$

(Year 1994)

(20 Marks)

Q21. Determine a family of curve for which the ratio of the y-intercept of the tangent to the radius vector is a constant.

(Year 1995)

(20 Marks)

Q22. Solve $(2x^2 + 3y^2 - 7)xdx + (3x^2 + 2y^2 - 8)ydy = 0$

(Year 1995)

(20 Marks)

Q23. Test whether the equation $(x + y)^2dx - (y^2 - 2xy - x^2)dy = 0$ is exact and hence solve it.

(Year 1995)

(20 Marks)

Q24. Solve $x^2 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(Year 1995)

(20 Marks)

Q25. Determine all real valued solution of the equations $y''' - iy'' + y' - iy = 0$,

$y' = \frac{dy}{dx}$

(Year 1995)

(20 Marks)

Q26. Find the solution of the equation $\frac{d^2y}{dx^2} + 4y = 8cos2x$, given that $y = 0, y' = 2$

When $x=0$.

(Year 1995)

(20 Marks)

Q27. Find the curves for which the sum of the reciprocals of the radius vector and polar sub tangent is constant.

(Year 1996)

(20 Marks)

Q28. Solve: $x^2(y - px) = yp^2, p \equiv \frac{dy}{dx}$

(Year 1996)

(20 Marks)

Q29. Solve: $ysin2xdx - (1 + y^2 + cos^2x)dy = 0$

(Year 1996)

(20 Marks)

Q30. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37sin3x = 0$. Find the value of y when $x = \frac{\pi}{2}$, if it is given that $y=3$ and $\frac{dy}{dx} = 0\ when\ x = 0$.

(Year 1996)

(20 Marks)

Q31. Solve $\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} = x^2 + 3e^{2x} + 4sinx$.

(Year 1996)

(20 Marks)

Q32. Solve: $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + logx$.

(Year 1996)

(20 Marks)

Q33. Solve the initial value problem $\frac{dy}{dx} = \frac{x^3}{x^2+y^2}, y(0) = 0$.

(Year 1997)

(20 Marks)

Q34. Solve $(x^2 - y^2 + 3x - y)dx + (x^2 - y^2 + x - 3y)dy = 0$.

(Year 1997)

(20 Marks)

Q35. Assume that the spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is *3mm*, and one hour later has been reduce to *2mm*. Find an expression for the radius of the rain drop at any time.

(Year 1997)

(20 Marks)

Q36. Solve $\frac{d^4y}{dx^4} + 6\frac{d^3y}{dx^3} + 11\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 20e^{-2x}sinx$.

(Year 1997)

(20 Marks)

Q37. Make use of the transformation $y(x) = u(x)secx$ to obtain the solution of

$y'' - 2y'tanx + 5y = 0, y(0) = 0, y'(0) = \sqrt{6}$.

(Year 1997)

(20 Marks)

Q38. Solve $(1 + 2x)^2 \frac{d^2y}{dx^2} - 6(1 + 2x) \frac{dy}{dx} + 16y = 8(1 + 2x)^2, y(0) = 0, y'(0) = 2.$

(Year 1997)

(20 Marks)

Q39. Solve the differential equation: $xy - \left(\frac{dy}{dx}\right) = y^3 e^{-x^2}$.

(Year 1998)

(20 Marks)

Q40. Show that the equation: $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents a family of hyperbolas having as asymptotes lines $x + y = 0, 2x + y + 1 = 0$.

(Year 1998)

(20 Marks)

Q41. Solve the differential equation: $y = 3px + 4p^2$.

(Year 1998)

(20 Marks)

Q42. Solve the differential equation: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}(x^2 + 9)$.

(Year 1998)

(20 Marks)

Q43. Solve the differential equation: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xsinx$.

(Year 1998)

(20 Marks)

Q44. Solve the differential equation: $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$.

(Year 1998)

(20 Marks)

Q45. Solve the differential equation: $\frac{xdx+ydy}{xdy-ydx} = \left(\frac{1-x^2-y^2}{x^2+y^2}\right)^{1/2}$.

(Year 1999)

(20 Marks)

Q46. Solve: $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + cosx.$

(Year 1999)

(20 Marks)

Q47. By the method of variation of parameters solve the differential equation

$\frac{d^2y}{dx^2} + a^2y = \sec(ax)$

(Year 1999)

(20 Marks)

Q48. Show that $3\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 8y = 0$ has an integral which is a polynomial in *x*. Deduce the general solution.

(Year 2000)

(12 Marks)

Q49. Reduce the $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R, where\ P, Q, R$ are functions *x* to the normal form. Hence, solve $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} sin2x$.

(Year 2000)

(15 Marks)

Q50. Solve the differential equation $y = x - 2ap + ap^2$. Find the singular solution and interpret it geometrically.

(Year 2000)

(15 Marks)

Q51. Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents a family of hyperbolas with a common axis and tangent at the vertex.

(Year 2000)

(15 Marks)

Q52. Solve $x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right)$ by the method of parameters.

(Year 2000)

(15 Marks)

Q53. A continuous function *y(t)* satisfies the differential equation

$\frac{dy}{dx} = \begin{cases} 1 + e^{1-t}, \ 0 \le t < 1 \\ 2 + 2t - 3t^2, 1 \le t < 5 \end{cases}$ if $y(0) = -e \ find \ y(2)$.

(Year 2001)

(12 Marks)

Q54. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log_e x$.

(Year 2001)

(12 Marks)

Q55. Solve $\frac{dy}{dx} + \frac{y}{x}\log_e y = \frac{y(\log_e y)^2}{x^2}$.

(Year 2001)

(15 Marks)

Q56. Find the general solution of $ayp^2 + (2x - b)p - y = 0, a > 0$.

(Year 2001)

(15 Marks)

Q57. Solve: $(D^2+1)^2 y = 24x\cos x$ given that $y = Dy = D^2 y = 0$ and $D^3 y = 12$ $when\ x = 0$.

(Year 2001)

(15 Marks)

Q58. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$.

(Year 2001)

(15 Marks)

Q59. Solve: $x\frac{dy}{dx} + 3y = x^3 y^2$.

(Year 2002)

(12 Marks)

Q60. Find the value of λ for which all solution of $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - \lambda y = 0$ tend to zero as $x \to \infty$.

(Year 2002)

(12 Marks)

Q61. Find the value of λ such that the following differential equation becomes exact.

$(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$. Further, for this value of λ, solve the equation.

(Year 2002)

(15 Marks)

Q62. Solve: $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$.

(Year 2002)

(15 Marks)

Q63. Using the method of variation of parameters, find the solutions of

$\frac{d^2y}{dx^2} - 2y\frac{dy}{dx} + y = xe^x sinx$ with $y(0) = 0\ and\ \left(\frac{dy}{dx}\right)_{x=0}$.

(Year 2002)

(15 Marks)

Q64. Solve: $(D-1)(D^2-2D+2)y = e^x\ where\ D \equiv \frac{d}{dx}$

(Year 2002)

(15 Marks)

Q65. Show that the orthogonal trajectory of a system of confocals ellipse is self orthogonal.

(Year 2003)

(12 Marks)

Q66. Solve $x\frac{dy}{dx} + ylogx = xye^x$.

(Year 2003)

(12 Marks)

Q67. Solve: $(D^2 - D) = 4(e^x + cosx + x^3), where\ D \equiv \frac{dy}{dx}$.

(Year 2003)

(15 Marks)

Q68. Solve the differential equation $(px^2+y^2)(px+y) = (P+1)^2, where\ p = \frac{dy}{dx}$ by reducing it to Clairaut's form using suitable subscriptions.

(Year 2003)

(15 Marks)

Q69. Solve: $(1-x^2)y'' + (1+x)y' + y = sin2[log(1+x)]$.

(Year 2003)

(15 Marks)

Q70. Solve the differential equation $x^2y'' - 4xy' + 6y = x^4sec^2x$ by the variation of parameters.

(Year 2003)

(15 Marks)

Q71. Find the solution of the following differential equation $\frac{dy}{dx} + ycosx = \frac{1}{2}sin2x$.

(Year 2004)

(12 Marks)

Q72. Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.

(Year 2004)

(12 Marks)

Q73. Solve: $(D^4 - 4D^2 - 5)y = e^x(x + cosx)$.

(Year 2004)

(15 Marks)

Q74. Reduce the equation $(px - q)(py + x) = 2p, where\ p = \frac{dy}{dx}$ to Clairaut's equation and hence solve it.

(Year 2004)

(15 Marks)

Q75. Solve: $(x + 2)\frac{d^2y}{dx^2} + (2x + 5)\frac{dy}{dx} + 2y = (x + 1)e^x$.

(Year 2004)

(15 Marks)

Q76. Solve the differential equation: $(1 - x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1 + x^2)y = x$.

(Year 2004)

(15 Marks)

Q77. Find the orthogonal trajectory of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter.

(Year 2005)

(12 Marks)

Q78. Solve: $xy\frac{dy}{dx} = \sqrt{(x^2 - y^2 - x^2y^2 - 1)}$

(Year 2005)

(12 Marks)

Q79. Solve the differential equation:

$[(x + 1)^4 D^3 + 2(x + 1)^3 D^2 - (x + 1)^2 D + (x + 1)]y = \frac{1}{x+1}$.

(Year 2005)

(15 Marks)

Q80. Solve the differential equation:

$(x^2 + y^2)(1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$ where $p = \frac{dy}{dx}$, by reducing it to Clairut's form by using suitable substitution.

(Year 2005)

(15 Marks)

Q81. Solve the differential equation $(sinx - xcosx)y'' - xsinx\ y' + ysinx = 0$ given that $y = sinx$ is a solution of this equation.

(Year 2005)

(15 Marks)

Q82. Solve the differential equation $x^2y'' - 2xy' + 2y = xlogx, x > 0$ by variation of parameters.

(Year 2005)

(15 Marks)

Q83. Find the family of curves whose tangents form an angle $\frac{\pi}{4}$ with the hyperbolas $xy = c, c > 0.$

(Year 2006)

(12 Marks)

Q84. Solve the differential equation $\left(xy^2 + e^{\frac{1}{x^3}}\right)dx - x^2ydy = 0.$

(Year 2006)

(12 Marks)

Q85. Solve: $(1 + y^2) + \left(x - e^{-\tan^{-1} y}\right)\frac{dy}{dx} = 0.$

(Year 2006)

(15 Marks)

Q86. Solve the equation: $x^2p^2 + py(2x + y) + y^2 = 0$ using the substitution *y*=*u* and *xy*=*v* find its singular solution, where $p = \frac{dy}{dx}$.

(Year 2006)

(15 Marks)

Q87. Solve the differential equation: $x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{y}{x} = 10\left(1 + \frac{1}{x^2}\right)$

(Year 2006)

(15 Marks)

Q88. Solve the differential equation: $(D^2 - 2D + 2)y = e^x tanx, D \equiv \frac{dy}{dx}$ by the method of variation of parameters.

(Year 2006)

(15 Marks)

Q89. Solve the ordinary equation $cos3x\frac{dy}{dx} - 3ysin3x = \frac{1}{2}sin6x + sin^3 3x, 0 < x < \frac{\pi}{2}$.

(Year 2007)

(12 Marks)

Q90. Find the solution of the equation $\frac{dy}{y} + xy^2 dx = -4xdx.$

(Year 2007)

(12 Marks)

Q91. Determine the general and singular solution of the equation

$y = x\frac{dy}{dx} + a\frac{dy}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$, a being a constant.

(Year 2007)

(15 Marks)

Q92. Obtain the general solution of $[D^3 - 6D^2 + 12D - 8]y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right), where\ D \equiv \frac{dy}{dx}$

(Year 2007)

(15 Marks)

Q93. Solve the equation $2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$.

(Year 2007)

(15 Marks)

Q94. Use the method of variation of parameters to find the general solution of the equation

$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x$.

(Year 2007)

(15 Marks)

Q95. Solve the differential equation $ydx + (x + x^3y^2)dy = 0$.

(Year 2008)

(12 Marks)

Q96. Use the method of variation of parameters to find the general solution of

$x^2y'' - 4xy' + 6y = -x^4 sinx.$

(Year 2008)

(12 Marks)

Q97. Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$,

$y(0) = 1, y'(0) = -1.$

(Year 2008)

(15 Marks)

Q98. Solve the differential equation $x^3y'' - 3x^2y' + xy = \sin(lnx) + 1.$

(Year 2008)

(15 Marks)

Q99. Solve the equation $y - 2xp + yp^2 = 0 \; where \; p = \frac{dy}{dx}.$

(Year 2008)

(15 Marks)

Q100. Find the Wronskian of the set of functions: $\{3x^3, |3x^3|\}$ on the interval $[-1,1]$ and determine whether the set is linearly dependent on $[-1,1]$.

(Year 2009)

(12 Marks)

Q101. Find the differential equation of the family of circles in the *xy-plane* passing through $(1,1) \; and \; (1,1)$.

(Year 2009)

(20 Marks)

Q102. Find the inverse Laplace transform of $F(s) = ln\left(\frac{s+1}{s+9}\right).$

(Year 2009)

(20 Marks)

Q103. Solve: $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2-x^2y-4y^3}, y(0) = 1.$

(Year 2009)

(20 Marks)

Q104. Consider the differential equation $y' = \alpha x, x > 0$ where α is a constant. Show that

(i) If $\emptyset(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
(ii) If $\alpha < 0$, then every solution tends to zero as $x \to \infty$.

(Year 2010)

(12 Marks)

Q105. Show that the differential equation $(3y^2 - x) + 2y(y^2 - 3)y' = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation.

(Year 2010)

(12 Marks)

Q106. Verify that $\frac{1}{2}(Mx + Ny)d[\log_e(xy)] + \frac{1}{2}(Mx - Ny)d\left[\log_e\left({}^{x}/_{y}\right)\right] = Mdx + Ndy.$
Hence show that

(i) If the differential equation $Mdx + Ndy = 0$ is a homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mdx + Ndy \equiv 0$;
(ii) If the differential equation $Mdx + Ndy = 0$ is not exact but is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$ then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx + Ny \equiv 0$.

(Year 2010)

(20 Marks)

Q107. Use the method of undermined coefficients to find the particular solutions of $y'' + y = sinx + (1 + x^2)e^x$ and hence find its general solution.

(Year 2010)

(20 Marks)

Q108. Obtain the solution of the ordinary differential equation $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$.

(Year 2011)

(10 Marks)

Q109. Determine the orthogonal trajectory of a family of curves represented by the polar equation $r = a(1 - cos\theta), (r, \theta)$ being the plane polar coordinates of any point.

(Year 2011)

(10 Marks)

Q110. Obtain Clairaut's form of differential equation $\left(x\frac{dy}{dx} - y\right)\left(y\frac{dy}{dx} + x\right) = a^2\frac{dy}{dx}$. Also find its general solution.

(Year 2011)

(15 Marks)

Q111. Obtain the general solution of the second order ordinary differential equation.

$y'' - 2y' + 2y = x + e^x cosx$, where dashes denotes the derivative *w.r.t. x*.

(Year 2011)

(15 Marks)

Q112. Using the method of variation of parameters, solve the second order differential equation $\frac{d^2y}{dx^2} + 4y = tan2x$.

(Year 2011)

(15 Marks)

Q113. Use the Laplace transform method to solve the following intial value problem:

$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, x(0) = 2 \; and \; \frac{dy}{dt}\Big|_{t=0} = -1.$

(Year 2011)

(15 Marks)

Q114. Solve : $\frac{dy}{dx} = \frac{2xye^{\left(x/y\right)^2}}{y^2\left(1+e^{\left(x/y\right)^2}\right)+2x^2e^{\left(x/y\right)^2}}$

(Year 2012)

(12 Marks)

Q115. Find the orthogonal trajectory of the family of curves $x^2 + y^2 = ax$.

(Year 2012)

(12 Marks)

Q116. Using Laplace transform, solve the initial value problem

$y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1.$

(Year 2012)

(12 Marks)

Q117. Show that the differential equation $(2xylogy)dx + \left(x^2 + y^2\sqrt{y^2+1}\right)dy = 0$ is not exact. Find an integrating factor and hence, the solution of the equation.

(Year 2012)

(20 Marks)

Q118. Find the general solution of the equation $y''' - y'' = -12x^2 + 6x$.

(Year 2012)

(20 Marks)

Q119. Solve the ordinary differential equation $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$.

(Year 2012)

(20 Marks)

Q120. If y is a function of x such that the differential coefficient $\frac{dy}{dx}$ is equal to

$\cos(x + y) + \sin(x + y)$. Find out a relation between x and y which is free from any derivative /differential.

(Year 2013)

(10 Marks)

Q121. Obtain the equation of the orthogonal trajectory of the family of the curves represented by $r'' = asinn\theta, (r, \theta)$ being the polar coordinates.

(Year 2013)

(10 Marks)

Q122. Solve the differential equation $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$.

(Year 2013)

(15 Marks)

Q123. Using the method of variation of parameters, solve the differential equation

$\frac{d^2y}{dx^2} + a^2y = secax.$

(Year 2013)

(15 Marks)

Q124. Find the general solution of the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = lnxsin(lnx)$.

(Year 2013)

(15 Marks)

Q125. By using Laplace transform method, solve the differential equation

$(D^2 + n^2)x = \mathrm{a}\sin(nt + \alpha)$, $D^2 = \frac{d^2}{dt^2}$ subject to initial conditions $x = 0, at\ t = 0$

In which a, n, α are constants.

(Year 2013)

(15 Marks)

Q126. Justify that a differential equation of the form:

$[y + xf(x^2 + y^2)]dx + [yf(x^2 + y^2) - x]dy = 0$ where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$.

(Year 2014)

(10 Marks)

Q127. Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency.

(Year 2014)

(10 Marks)

Q128. Solve by the method of variation of parameters: $\frac{dy}{dx} - 5y = sinx$.

(Year 2014)

(10 Marks)

Q129. Solve the differential equation: $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65cos(\log_e x)$.

(Year 2014)

(20 Marks)

Q130. Solve the following differential equation:

$x\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$, where e^{2x} is a solution to its corresponding homogeneous differential equation.

(Year 2014)

(15 Marks)

Q131. Find the sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$, to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation of $(x^2 + xy)dx + (y^2 + xy)dy = 0$ and solve it.

(Year 2014)

(15 Marks)

Q132. Solve the initial value problem $\frac{d^2y}{dt^2} + y = 8e^{-2t} sint, y(0) = 0, y'(0) = 0$ by using Laplace transform.

(Year 2014)

(20 Marks)

Q133. Solve the differential equation: $xcosx\frac{dy}{dx} + y(xsinx + cosx) = 1$.

(Year 2015)

(10 Marks)

Q134. Solve the differential equation:

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$$

(Year 2015)

(10 Marks)

Q135. Find the constant a so that $(x+y)^a$ is the integrating factor of $(4x^2+2xy+6y)dx+(2x^2+9y+3x)dy=0$ and hence solve the differential equation.

(Year 2015)

(12 Marks)

Q136. (i) obtain the Laplace inverse transform of $\left\{ln\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-5s}\right\}$

(ii) Using Laplace transform, solve $y''+y=t, y(0)=1, y'(0)=-2$

(Year 2015)

(6+6=12 Marks)

Q137. Solve the differential equation $x=py-p^2$ where $p=\frac{dy}{dx}$.

(Year 2015)

(13 Marks)

Q138. Solve $x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x)$.

(Year 2015)

(13 Marks)

Q139. Find a particular integral of $\frac{d^2y}{dx^2}+y=e^{x/2}\sin\frac{x\sqrt{3}}{2}$.

(Year 2016)

(10 Marks)

Q140. Show that the family of parabola, $y^2=4cx+4c^2$ is self orthogonal.

(Year 2016)

(10 Marks)

Q141. Solve $\{y(1-xtanx)+x^2cosx\}dx-xdy=0$.

(Year 2016)

(10 Marks)

Q142. Using the method of variation of parameters solve the differential equation

$(D^2 + 2D + 1)y = e^{-x} \log(x), \left[D = \frac{d}{dx}\right].$

(Year 2016)

(15 Marks)

Q143. Find the general solution of the equation $x^2 \frac{d^3y}{dx^3} - 4x \frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4.$

(Year 2016)

(15 Marks)

Q144. Using Laplace transformation solve the following equation:

$y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6.$

(Year 2016)

(10 Marks)

Q145. Find the differential equation representing the entire circle in the *xy-plane* .

(Year 2017)

(10 Marks)

Q146. Solve the following simultaneous linear differential equations:

$(D + 1)y = z + e^x$ and $(D + 1)z = y + e^x$ where *y* and *z* are functions of

independent variable $x \text{ and } D = \frac{d}{dx}$.

(Year 2017)

(8Marks)

Q147. If the growth rate of the population of bacteria at time *t* is proportional to the amount present at the time and population doubles in one week, then how much bacteria's can be expected after 4 weeks?

(Year 2017)

(8 Marks)

Q148. Consider the differential equation $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ where $p = \frac{d}{dx}$ substituting $u = x^2$ *and* $v = y^2$. Reduce the equation to Clairut's form in terms of u, v *and* $p' = \frac{dv}{du}$ hence otherwise solve the equation.

(Year 2017)

(10 Marks)

Q149. Solve the following initial value differential equations $20y'' + 4y' + y = 0$, $y(0) = 3.2, y'(0) = 0$.

(Year 2017)

(7 Marks)

Q150. Solve the differential equation: $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin(x^2)$.

(Year 2017)

(9 Marks)

Q151. Solve the following differential equation using method of variation of parameters $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2$.

(Year 2017)

(8 Marks)

Q152. Solve the following initial value problem using Laplace Transform:

$$\frac{d^2y}{dx^2} + 9y = r(x), y(0) = 0, y'(0) = 4 \text{ where } r(x) = \begin{cases} 8sinx \;\; if\ 0 < x < \pi \\ \quad 0 \;\; if\ x \geq \pi \end{cases}$$

(Year 2017)

(17 Marks)

Q153. Solve: $\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$

(Year 2018)

(13 Marks)

Q154. Solve: $y'' + 16y = 32sec2x$

(Year 2018)

(13 Marks)

Q155. Solve : $(1+x)^2y'' + (1+x)y' + y = 4\cos(\log(1+x))$

(Year 2018)

(13 Marks)

Q156. Solve the initial value problem:

$$y'' - 5y' + 4y = e^{2t}\ at\ y(0) = \frac{19}{12}, y'(0) = \frac{8}{3}$$

(Year 2018)

(13 Marks)

Q157. Find $\alpha\ and\ \beta$ such that $x^\alpha y^\beta$ is an integrating factor of

$(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation.

(Year 2018)

(12 Marks)

Q158. Find *f(y)* such that $(2xe^y + 3y^2)dy + \left(3x^2 + f(y)\right)dx = 0$ is exact and hence solve it.

(Year 2018)

(12 Marks)

Q159. Solve: (i) $y'' - y = x^2 e^{2x}$

(ii) $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$

(Year 2018)

(10+10=20 Marks)

Q160. Determine the complete solution of the differential equation:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2 sin2x$$

(Year 2019)

(10 Marks)

Q161. Obtain the singular solution of the differential solution of the differential equation $\left(\frac{dy}{dx}\right)^2 \left(\frac{y}{x}\right)^2 cot^2\alpha - 2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 cosec^2\alpha = 1$. Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution.

(Year 2019)

(15 Marks)

Q162. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y'' - 2xy + 2y = x^3 sinx$ and then find the general solution of the given equation by the method of variation of parameters.

(Year 2019)

(15 Marks)

Q163. Solve the differential equation:

(i) $$\frac{d^2y}{dx^2} + (3sinx - cotx)\frac{dy}{dx} + 2ysin^2x = e^{-x}sin^2x.$$

(ii) $$(2ysinx + 3y^4sinxcosx)dx - (4y^3cos^2x + cosx)dy = 0$$

(Year 2019)

(10+10=20 Marks)

CIVIL SERVICES

PREVIOUS YEAR QUESTIONS

SEGMENT- WISE

ORDINARY DIFFERENTIAL EQUATIONS

1. Show that the general solution of the differential equation $\frac{dy}{dx} + Py = Q$ can be written in the form $y = \frac{Q}{P} - e^{-\int P dx}\left\{C + \int e^{\int P dx} d\left(\frac{Q}{P}\right)\right\}$, where P, Q are non-zero functions of x and C, an arbitrary constant. **[2022][10]**
2. Show that the orthogonal trajectories of the system of parabolas: $x^2 = 4a(y + a)$ belong to the same system. **[2022][10]**
3. Solve the following differential equation by using the method of variation of parameters: $(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = (x^2 - 1)^2$, given that $y = x$ is one solution of the reduced equation. **[2022][15]**
4. Solve the following initial value problem by using Laplace's transformation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = h(t)$, where $h(t) = \begin{cases} 2, & 0 < t < 4, \\ 0, & t > 4, \end{cases}$ $y(0) = 0,\ \ y'(0) = 0$ **[2022][15]**
5. **(i)** Find the general and singular solutions of the differential equation: $(x^2 - a^2)p^2 - 2xyp + y^2 + a^2 = 0$, where $p = \frac{dy}{dx}$. Also give the geometric relation between the general and singular solutions.
   (ii) Solve the following differential equation:
   $(3x + 2)^2\frac{d^2y}{dx^2} + 5(3x + 2)\frac{dy}{dx} - 3y = x^2 + x + 1$ **[2022][10+10]**
6. Solve the differential equation:
   $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x\cos 2x$ **[2021][10]**
7. Solve the initial value problem:
   $\frac{d^2y}{dx^2} + 4y = e^{-2x}\sin 2x; y(0) = y'(0) = 0$
   using Laplace transform method. **[2021][10]**

8. Solve the equation: $\frac{d^2y}{dx^2} + (\tan x - 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x$ completely by demonstrating all the steps involved. **[2021][15]**
9. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1; a > b > 0$ are constants and λ is a parameter. Show that the given family of curves is self orthogonal. **[2021][10]**

10. Find the general solution of the differential equation:

$$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = 0.$$

Hence, solve the differential equation: $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$ by the method of variation of parameters. **[2021][10]**

11. Using the method of variation of parameters, solve the differential equation $y'' + (1 - \cot x)y' - y\cot x = \sin^2 x$, if $y = e^{-x}$ is one solution of CF. **[2020][20]**

12. Using Laplace transform, solve the initial value problem $ty'' + 2ty' + 2y = 2$; $y(0) = 1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution? **[2020][10]**

13. Find the general and singular solutions of the differential equation $9p^2(2-y)^2 = 4(3-y)$, where $p = \frac{dy}{dx}$. **[2020][10]**

14. Solve the following differential equation:

$(x+1)^2y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1)$ **[2020][10]**