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EXADEMY

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LINEAR ALGEBRA

Q1. Prove that the characteristic roots of a Hermitian matrix are all real and a characteristic root of a skew-Hermitian is either zero or a pure imaginary number.

(Year 1992)

(20 Marks)

Q2. Transform the following to diagonal forms and give the transformational employed:

$x^2 + 2y, 8x^2 - 4xy + 5y^2$

(Year 1992)

(20 Marks)

Q3. State Cayley-Hamilton theorem and use it to calculate the inverse of the matrix

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

(Year 1992)

(20 Marks)

Q4. Prove that a necessary and sufficient condition of a real quadratic from X' AX to be positive definite is that the leading principal minors of A are all positive.

(Year 1992)

(20 Marks)

Q5. For what values of η do the following equations

$x + y + z = 1$

$x + 2y + 4z = \eta$

$x + 4y + 10z = \eta^2$, Have solutions ? Solve then completely in each case.

(Year 1992)

(20 Marks)

Q6. Let T: $M_{2,1} \to M_{2,3}$ be a linear transformation defined by (with usual notations)

$$T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2 & 1 & 3\\4 & 1 & 5\end{pmatrix}, T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}6 & 1 & 0\\0 & 0 & 2\end{pmatrix} \text{ Find } T\begin{pmatrix}x\\y\end{pmatrix}$$

(Year 1992)

(20 Marks)

Q7. Verify which of the following are linear transformations?

I. T: $R \to R^2$ defined by T(x) = (2x, -x)
II. T: $R^2 \to R^3$ defined by T(x) = (xy, y, x)
III. T: $R^2 \to R^3$ defined by T(x) = (x + y, y, x)
IV. T: $R \to R^2$ defined by T(x) = (1, -1)

(Year 1992)
(20 Marks)

Q8. Let S = {(x, y, z) / x + y + z = 0}, x, y, z being real. Prove that S is a subspace of R^3. Find a basis of S.

(Year 1992)

(20 Marks)

Q9. Let V and U be vector spaces over the field K and let V be of finite dimension. Let $T : V \rightarrow U$ be a linear map. $\dim V = \dim R(T) + \dim N(T)$

(Year 1992)

(20 Marks)

Q10. Determine the following form as definite, semi-definite or indefinite:

$2x_1^2 + 2x_2^2 + 3x_3^2 - 4x_2x_3 - 4x_1x_3 + 2x_1x_2$

(Year 1993)

(20 Marks)

Q11. Find the rank of the matrix given below by reducing to canonical form.

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

(Year 1993)

(20 Marks)

Q12. A matrix B of order n x n is of the form λA where λ is a scalar and A has unit elements everywhere except in the diagonal which has elements μ. Find λ and μ so that B may be orthogonal.

(Year 1993)

(20 Marks)

Q13. Show that any two Eigen vectors corresponding to two distinct Eigen values of

I. Hermitian Matrix
II. Unitary matrix are orthogonal

(Year 1993)

(20 Marks)

Q14. If *A* be an orthogonal matrix with the property that -1 is not an Eigen value, then show that *a* is expressed as $(I - S)(S + S)^{-S1}$ for some suitable skew-symmetric matrix *S.*

(Year 1993)

(20 Marks)

Q15. Prove that the inverse of $\begin{bmatrix} A & O \\ B & C \end{bmatrix}$ is $\begin{bmatrix} A^{-1} & O \\ C^{-1}BA^{-1} & C^{-1} \end{bmatrix}$ where A, C are non-singular matrices and hence find the inverse of:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(Year 1993)

(20 Marks)

Q16. If the matrix of a linear operator T on R^2 relative to the standard basis {(1, 0), (0, 10)} is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, what is the matrix of T relative to the basis B = {(1, 1), (1, -1)}?

(Year 1993)

(20 Marks)

Q17. Define rank and nullity of a linear transformation T. if V be a finite dimensional vector space and T a linear operator on V such that rank T^2 = rankT, then prove that the null space of T = the null space of T^2 and the intersection of the range space and null space to T is the zero subspace of V

(Year 1993)

(20 Marks)

Q18. Show that the set S = {(1, 0, 0), (1, 1, 0), (1, 1, 1), (0, 1, 0)} spans the vector space R^3 (R) but it is not a basis set.

(Year 1993)

(20 Marks)

Q19. Reduce the following symmetric matrix to a diagonal form and interpret the result in terms of quadratic forms: A = $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$

(Year 1994)

(20 Marks)

Q20. Show that a matrix congruent to a skew-symmetric matrix is skew-symmetric. Use the result to prove that the determinant of skew-symmetric matrix of even order is the square of a rational function of its element.

(Year 1994)

(20 Marks)

Q21. Prove that the Eigen vectors corresponding to the distinct Eigen values of a square matrix are linearly independent.

(Year 1994)

(20 Marks)

Q22. Show that f_1 (t) = 1, f_2 (t) = t – 2, f_3 (t) = $(t – 2)^2$ form a basis of P_3, the space of polynomials with degree ≤ 2. Express $3t^2 – 5t + 4$ as a linear combination of f_1, f_2, f_3.

(Year 1994)

(20 Marks)

Q23. Let A and for every. Show that A is a non-singular matrix. Hence or otherwise prove that the Eigen values of A lie in the discs in the complex plane.

(Year 1995)

(20 Marks)

Q24. Let A and B be square matrices of order n. Show that AB – BA can never be equal to unit matrix.

(Year 1995)

(20 Marks)

Q25. Let A be a symmetric matrix. Show that A is positive definite if and only if its Eigen values are all positive.

(Year 1995)

(20 Marks)

Q26. If a and b complex numbers such that and H is a Hermitian matrix, show that the Eigen values of lie on a straight line in the complex plane.

(Year 1995)

(20 Marks)

Q27. Let A and B be matrices of order 'n'. Prove that if (I – AB) is invertible, then (I – AB) is also invertible and $(I - AB)^{-1} = I + B(I - AB)^{-1} A$. Show that AB and BA have precisely the same characteristic.

(Year 1995)

(20 Marks)

Q28. Show that $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is diagonalizable and hence determine A^5.

(Year 1995)

(20 Marks)

Q29. Define a similar matrix. Prove that the characteristic equation of two similar matrices is the same. Let 1, 2, and 3 be the Eigen – values of a matrix. Write down such a matrix. Is such a matrix unique ?

(Year 1995)

(20 Marks)

Q30. Let A be a square matrix of order 'n'. Prove that AX = b has solution if and only if $b \in R^n$ is orthogonal to all solutions Y of the system $A^T Y = 0$

(Year 1995)

(20 Marks)

Q31. Let T be the linear operator in R^3 defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$. What is the matrix of T in the standard ordered basis of R^3 ? What is a basis of range space of T and a basis of null space of T ?

(Year 1995)

(20 Marks)

Q32. Reduce to canonical form the orthogonal matrix $\begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$

(Year 1996)

(20 Marks)

Q33. Let A and B be n x n matrices such that AB = BA. Show that A and B have a common characteristic vector.

(Year 1996)

(20 Marks)

Q34. Find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$ by computing its characteristic polynomial.

(Year 1996)

(20 Marks)

Q35. Solve

$x + y - 2z = 1$

$2x - 7z = 3$

$x + y - z = 5$ by using Cramer's Rule.

(Year 1996)

(20 Marks)

Q36. Let V and W be finite dimensional vector spaces such that $\dim V \geq \dim W$. Show that there is always a linear map from V onto W.

(Year 1996)

(20 Marks)

Q37. Let $V = R^3$ and $T : V \rightarrow V$ be linear map defined by $T(x, y, z) = (x + z. -2x + y, -x + 2y + z)$. What is the matrix of T with respect to the basis (1, 0, 1), (-1, 1, 1) and (0, 1, 1) ? Using this matrix, write down the matrix of T with respect to the basis (0, 1, 2), (-1, 1, 1) and (0, 1, 1)

(Year 1996)

(20 Marks)

Q38. Let $V = R^3$ and v_1, v_2, v_3 be a basis of R^3. Let $T : V \rightarrow V$ be a linear transformation such that by writing the matrix of T with respect to another basis, show that the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(Year 1996)

(20 Marks)

Q39. Let V be a finite dimensional vector space and $v \in V$, $v \neq 0$. Show that there exists a linear functional 'f' on V.

(Year 1996)

(20 Marks)

Q40. Let W_1 be the space generated by (1, 1, 0, -1), (2, 6, 0) and (-2, -3, -3, 1) and let W_2 be the space generated by (-1, -2, -2, 2), (4, 6, 4, -6) and (1, 3, 4, -3). Find a basis for the space $W_1 + W_2$.

(Year 1996)

(20 Marks)

Q41. Find an invertible matrix P which reduces $Q(x, y, z) = 2xy + 2yz + 2zx$ to its canonical form.

(Year 1997)

(20 Marks)

Q42. Find the characteristics roots and their corresponding vectors for the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(Year 1997)

(20 Marks)

Q43. Define a positive definite matrix. Show that a positive definite matrix is always non – singular. Prove that its converse does not hold.

(Year 1997)

(20 Marks)

Q44. Let A = [a_{ij}] be a square matrix of order n such that [a_{ij}] ≤ M for all i, j = 1, 2, …n. Let λ be an Eigen-value of A. Show that |λ| ≤ nM.

(Year 1997)

(20 Marks)

Q45. Show that $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$ is diagonalizable over R and find a matrix P such that $P^{-1}AP$ is diagonal. Hence determine A^{25}.

(Year 1997)

(20 Marks)

Q46. Let a square matrix A of order 'n' be such that each of its diagonal elements is μ and each of its off diagonal elements is 1. If B = λA is orthogonal, determine the value of λ and μ.

(Year 1997)

(20 Marks)

Q47. Let V be the vector space of 2x2 matrices over R. Determine whether the matrices A, B, C ϵ V are dependent where $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -5 \\ -4 & 0 \end{bmatrix}$

(Year 1997)

(20 Marks)

Q48. Verify that the transformation defined by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ is a linear transformation from R^2 into R^3. Find its range, null space and nullity.

(Year 1997)

(20 Marks)

Q49. Let V be the vector space of polynomials over R. Find a basis and dimension of the subspace W of V spanned by the polynomials

$v_1 = t^3 - 2t^2 + 4t + 1$, $v_2 = 2t^3 - 3t^2 + 9t - 1$, $v_3 = t^3 + 6t^2 - 5$, $v_4 = 2t^3 - 5t^2 + 7t + 5$

(Year 1997)

(20 Marks)

Q50. Reduce to diagonal matrix by rational congruent transformation the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

(Year 1998)

(20 Marks)

Q51. Find all real 2x2 matrices A whose characteristic roots are real and which satisfy AA' = 1

(Year 1998)

(20 Marks)

Q52. Let A be a matrix. Then show that the sum of rank and nullity of A is n

(Year 1998)

(20 Marks)

Q53. Prove that a necessary and sufficient condition for a nxn real matrix to be similar to a diagonal matrix A is that the set of characteristic vectors A of includes a set of linearly independent vector.

(Year 1998)

(20 Marks)

Q54. If T is a complex matrix of order 2x2 such that $trT = trT^2 = 0$, then show that $T^2 = 0$.

(Year 1998)

(20 Marks)

Q55. If A and B are two matrices of order 2x2 such that A is skew Hermitian and AB = B then show that B = 0.

(Year 1998)

(20 Marks)

Q56. Let $T : R^3 \rightarrow R^3$ be defined by $T(x_1, x_2, x_3) = (x_2, x_3 - cx_1bx_2 - ax_3)$ where a, b, c are fixed real numbers. Show that T is a linear transformation of R^3 and that $A^3 + aA^2 + ba = cI = 0$ where A is the matrix of T with respect to standard basis of R^3.

(Year 1998)

(20 Marks)

Q57. If is a finite dimensional vector space over R and if and are two linear transformations from V to R such that Vf(v) = 0 in piles g(v) = 0, then prove that $g = \lambda f$ form some λ in R.

(Year 1998)

(20 Marks)

Q58. Given two linearly independent vectors (1, 0, 1, 0) and (0, -1, 1) of R^4 find a basis of which included there two vectors.

(Year 1998)

(20 Marks)

Q59. Test for the positive definiteness of the quadratic form $2x^2 + y^2 + 2z^2 - 2xz$.

(Year 1999)

(20 Marks)

Q60. If A is a skew symmetric matrix of order n. Prove that $(I - A)(I + A)^{-1}$ is orthogonal.

(Year 1999)

(20 Marks)

Q61. Test for congruency of the matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$. Prove that $A^{2n} = B^{2m}I$ when and are positive integers.

(Year 1999)

(20 Marks)

Q62. Diagonalize the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$

(Year 1999)

(20 Marks)

Q63. If the matrix of a linear transformation T on $V_2(R)$ with respect to the basis, then what is the matrix of with respect to the ordered basis B = {(1, 0), (0, 1)} is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then what is the matrix of T with respect to the ordered basis.

(Year 1999)

(20 Marks)

Q64. Let V be the vector space of functions from R to R (the real numbers). Show that f, g, h, in V are linearly independent where $f(t) = e^{2t}$, $g(t) = t^2$ and $h(t) = t$.

(Year 1999)

(20 Marks)

Q65. Reduce the equation $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx + x - y - 2z + 6 = 0$ into canonical form and determine the nature of the quadratic.

(Year 2000)

(15 Marks)

Q66. Prove that two similar matrices have the same characteristic roots. Is its converse true ? Justify your claim.

(Year 2000)

(15 Marks)

Q67. Prove that a system AX = B if non- homogeneous equations in unknowns have a unique solution provided the coefficient matrix is non-singular.

(Year 2000)

(15 Marks)

Q68. Prove that a real symmetric matrix A is positive definite if and only A = BB’ if for some non-singular matrix B. Also show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 11 \end{bmatrix}$ is positive definite and find the matrix B such that A = BB’

(Year 2000)

(15 Marks)

Q69. Show that if λ is a characteristic root of a non-singular matrix A then λ^{-1} is a characteristic root of A^{-1}.

(Year 2000)

(15 Marks)

Q70. Let V be a vector space over R and T = {(x, y)|x, y ϵ v}. Define addition in component wise and scalar multiplication by complex number $\alpha + i\beta$ by $(\alpha + i\beta)(x, y) = (\alpha x + \beta y, \beta y + \alpha y)$ for all $\alpha\beta$ ϵ R. Show that T is a vector space over C.

(Year 2000)

(12 Marks)

Q.71. Show that the real quadratic form $\phi = n(x_1^2 + x_2^2 + \ldots + x_n^2) - (x_1x_2 + \ldots + x_n)^2$ in n variables is positive semi-definite.

(Year 2001)

(15 Marks)

Q72. Determine an orthogonal matrix P such that it is a diagonal, where = $\begin{bmatrix} 7 & 4 & 4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$

(Year 2001)

(15 Marks)

Q73. When is a square matrix A said to be congruent to a square matrix B ? Prove that every matrix congruent to skew-symmetric matrix is skew symmetric.

(Year 2001)

(15 Marks)

Q74. If A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$. Hence determine A^{50}.

(Year 2001)

(15 Marks)

Q75. If λ is a characteristic root of a non-singular matrix A then prove that $|A| / \lambda$ is a characteristic root of Adj.A

(Year 2001)

(12 Marks)

Q76. Show that the vectors (1, 0, -1), (0, -3, 2) and (1, 2, 1) form a basis for the vector space $R^3(R)$.

(Year 2001)

(12 Marks)

Q77. Use Cayley – Hamilton theorem to find the inverse of the following matrix: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

(Year 2002)

(15 Marks)

Q78. Solve the following system of linear equations:

$x_1 - 2x_2 - 3x_3 + 4x_4 = -1$

$-x_1 + 3x_2 + 5x_3 - 5x_4 - 2x_4 = 0$

$2x_1 + x_2 - 2x_3 + 3x_4 - 4x_5 = 17$

(Year 2002)

(15 Marks)

Q79. Let A be a real 3x3 symmetric matrix with Eigen values 0, 0 and 5. If the corresponding Eigen-vectors are (2, 0, 1), (2, 1, 1) and (1, 0, -2) then find the matrix A.

(Year 2002)

(15 Marks)

Q80. Let $R^5 \rightarrow R^5$ be a linear mapping given by T(a, b, c, d, e) = (b – d, +e, b, 2d + e, b + e). Obtain based for its null space and range space.

(Year 2002)

(15 Marks)

Q81. A square matrix A is non-singular if and only if the constant term in its characteristic polynomial is different from zero.

(Year 2002)

(12 Marks)

Q82. Show that the mapping $T : R^3 \rightarrow R^3$ where $T(a, b, c) = (a - b, d - c, a + c)$ is linear and non singular.

(Year 2002)

(12 Marks)

Q83. Reduce the quadratic form given below to canonical form and find its rank and signature

$x^2 + 4y^2 + 9z^2 + u^2 - 12yz + 6zx - 4xy - 2xu - 6zu$

(Year 2003)

(15 Marks)

Q84. If A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ the find a diagonal matrix D and a matrix B such that A = BDB’ where B’ denotes the transpose of B.

(Year 2003)

(15 Marks)

Q85. If H is a Hermitian matrix, then show that $A = (H + iI)^{-1} (H - iI)$ is a unitary matrix. Also show that every unitary matrix can be expressed in this form, provided 1 is not an Eigen value of A.

(Year 2003)

(15 Marks)

Q86. Prove that the Eigen vectors corresponding to distinct Eigen values of a square matrix are linearly independent.

(Year 2003)

(15 Marks)

Q87. if = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then find the matrix represented by

$$2A^{10} - 10A^{9} + 14A^{8} - 6A^{7} - 3A^{6} + 15A^{4} - 21A^{4} + 9A^{3} + A - 1$$

(Year 2003)

(12 Marks)

Q88. Let S be any non-empty subset of a vector pace V over the field F. Show that the set $\{a_1\alpha_1 + a_2\alpha_2 + \ldots + a_n\alpha_n : a_1, a_2, \ldots, a_n \in \alpha_1, \alpha_2, \ldots, \alpha_n \in S, n \in H\}$ is the subspace generated by S.

(Year 2003)

(12 Marks)

Q89. Define a positive definite quadratic for, Reduce the quadratic form to canonical form. Is this quadratic form positive definite ?

(Year 2004)

(15 Marks)

Q90 Find the characteristic polynomial of the matrix A = $\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ Hence find A^{-1} and A^{6}

(Year 2004)

(15 Marks)

Q91. Verify whether the following system of equations is consistent or not

$-2x + 5y - z = 0$

$-x + 4y + z = 4$

(Year 2004)

(15 Marks)

Q92. Show that the linear transformation from IR^3 to IR^4 which is represented by the matrix

$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is one-to-one. Find a basis for its image.

(Year 2004)

(12 Marks)

Q93. Show that $f : R^3 \rightarrow IR$ is a linear transformation, where $f(x, y, z) = 3x + y - z$ what is the dimension of the kernel ? Find a basis for the kernel.

(Year 2004)

(12 Marks)

Q94. Let S be space generated by the vectors {(0, 2, 6), (3, 1, 6), (4, -2, -2)} what is the dimension of the space S ? Find a basis for S

(Year 2004)

(12 Marks)

Q95. Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ to the sum of squares. Also find the corresponding linear transformation, index and signature.

(Year 2005)

(15 Marks)

Q96. If S is a skew-Hermitian matrix, then show that is a unitary matrix. Also show that $A = (I + S)(I - S)^{-1}$ every unitary matrix can be expressed in the above form provided -1 is not a Eigen value of A.

(Year 2005)

(15 Marks)

Q97. Find the inverse of the matrix given below using elementary row operations only:

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

(Year 2005)

(15 Marks)

Q98. Let T be a linear transformation of R^3 whose matrix relative to the standard basis of R^3 is

$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ find the matrix of T relative to the basis $\beta = \{(1, 1, 1), (1, 1, 0), (0, 1, 1)\}$

(Year 2005)

(15 Marks)

Q99. Let V be the vector space of polynomials in x degrees $\leq n$ over R. Prove that the set $\{1, x, x^2, \ldots, x^n\}$ is a basis for the set of all polynomials in x.

(Year 2005)

(12 Marks)

Q100. Find the values of k for which the vectors (1, 1, 1, 1), (1, 3, -2, k), (2, 2k – 2, -k – 2, 3k – 1) and (3, k + 2, -3, 2k + 1) are linearly independent in R^4

(Year 2005)

(12 Marks)

Q101. Find the quadratic form q(x, y) corresponding to the symmetric matrix $A = \begin{bmatrix} 5 & -3 \\ -3 & 8 \end{bmatrix}$ Is this quadratic form positive definite ? Justify your answer.

(Year 2006)

(15 Marks)

Q102. Investigate the values of λ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have

I. No solution
II. Unique solution
III. An infinite number of solutions

(Year 2006)

(15 Marks)

Q103. Using elementary row operations, find the rank of the matrix

$$\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

(Year 2006)

(15 Marks)

Q104. If $T : R^2 \to R^2$ is defined by $T(x, y) = (2x - 3y, x + y)$ compute the matrix of T relative to the basis $\beta\{(1, 2), (2, 3)\}$

(Year 2006)

(15 Marks)

Q105. State Cayley – Hamilton theorem and using it, find the inverse of $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(Year 2006)

(12 Marks)

Q106. Let V be the vector space of all 2x2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V.

(Year 2006)

(12 Marks)

Q107. Let S be the vector space of all polynomials, p(x) with real coefficients, of degree less than or equal to two considered over the real field R such that p(0) and p(1) = 0. Determine a basis for S and hence its dimension.

(Year 2007)

(12 Marks)

Q108. Let T be the linear transformation from R^3 to R^4 defined by $T(x_1, x_2, x_3) = (2x_1 + x_2 + x_3x_1, x_2x_1 + x_3, 3x_1 + x_2 = 2x_3)$ for each $(x_1, x_2, x_3) \in R$. determine a basis for the Null space of T. What is the dimension of the Range space of T ?

(Year 2007)

(12 Marks)

Q109. Let W be the set of all 3x3 symmetric matrices over R does it form a subspace of the vector space of the 3x3 matrices over R ? In case it does, construct a basis for this space and determine its dimensions.

(Year 2007)

(15 Marks)

Q110 Consider the vector space X = {p(x)} is a polynomials degree less than or equal to 3 with real coefficients. Over the real field R define the map $D : X \rightarrow X$ by $(Dp)(x) := P_1 + 2P_2x + 2P_3x^2$ where $p(x) = P_0 + P_1x + P_2x^2 + P_3x^3$ is D a linear transformation on X ? If it is then construct the matrix representation for D with respect to the order basis $\{1, x, x^2, x^3\}$ for X.

(Year 2007)

(12 Marks)

Q111. Reduce the quadratic form $q(x, y, z) = x^2 + 2y^2 - 4xz - 4yz + 7z^2$ to canonical form. Is it positive definite ?

(Year 2007)

(12 Marks)

Q112. Show that the matrix A is invertible if and only if the adj(A) is invertible. Hence find $|adj(A)|$

(Year 2008)

(12 Marks)

Q113 Let S be a non-empty set and let V denote the set of all functions from S into R. Show that V is vector space with respect to the vector addition $(f + g)(x) = f(x) + g(x)$ and scalar multiplication $(c.f)(x) = cf(x)$

(Year 2008)

(12 Marks)

Q114. Show that B = {(1, 0, 0), (1, 1, 0), (1, 1, 1)} is a basis of R^3. Let T: $R^3 \rightarrow R^3$ be a linear transformation such that T(1, 0, 0) = (1, 0, 0), T(1, 1, 0) = (1, 1, 1) and T(1, 1, 1) = (1, 1, 0). Find T(x, y, z)

(Year 2008)

(15 Marks)

Q115. Let A be a non-singular matrix. Show that if $I + A + A^2 + \ldots + A^n = 0$, then $A^{-1} = A^n$

(Year 2008)

(15 Marks)

Q116. Find the dimension of the subspace of R^4 spanned by the set {(1, 0, 0, 0) (0, 1, 0, 0) (1, 2, 0, 1) (0, 0, 0, 1)}. Hence find a basis for the subspace.

(Year 2008)

(15 Marks)

Q117. Find a Hermitian and skew-hermitian matrix each whose sum is the matrix

$$\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2+3i & 2 \\ -i+1 & 4 & 5i \end{bmatrix}$$

(Year 2009)

(12 Marks)

Q118. Prove that the set V of the vectors (x_1, x_2, x_3, x_4) in which R^4 satisfy the equation $x_1 + x_2 + x_3 + x_4 = 0$ and $2x_1 + 3x_2 - x_3 + x_4 = 0$ is a subspace of R^4. What is the dimension of this subspace? Find one of its bases.

(Year 2009)

(12 Marks)

Q119. Let β = {(1, 1, 0) (1, 0, 1), (0, 1, 1)} and β' = {(2, 1), (1, 2, 1), (-1, 1, 1)} be the two ordered bases of R^3. Then find a matrix representing the linear transformation T : $R^3 \rightarrow R^3$ which transforms β into β'. Use this matrix representation to find T(x), where x = (2, 3, 1).

(Year 2009)

(20 Marks)

Q120. Find a 2x2 real matrix A which both orthogonal and skew-symmetric. Can there exist a 3x3 real matrix for which both orthogonal and skew-symmetric? Justify your answer.

(Year 2009)

(20 Marks)

Q121. Let L: $R^4 \rightarrow R^3$ be a linear transformation defined by L = (x_1, x_2, x_3, x_4) = ($x_3 + x_4 - x_1 - x_2$, $x_3 - x_2$, $x_4 - x_1$). Then find the rank and nullity of L. Also, determine null space and range space of L.

(Year 2009)

(20 Marks)

Q122. Prove that the set V of all 3x3 real symmetric matrices forms a linear subspace of the space of all 3x3 real matrices. What is the dimension of this subspace ? Find at least of the bases for V.

(Year 2009)

(20 Marks)

Q123. If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the Eigen values of matrix A = $\begin{bmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 44 & 2 & 28 \end{bmatrix}$ show that

$$\sqrt{\lambda 1^2 + \lambda 2^2 + \lambda 3^2} \leq \sqrt{1949}$$

(Year 2010)

(12 Marks)

Q124. What is the null space of the differential transformation d/dx : $p_n \rightarrow p_n$ is the space of all polynomials of degree ≤ -n over the real numbers ? What is the null space of the second derivatives as a transformation of ? What is the null space of the kth derivative p_n ?

(Year 2010)

(12 Marks)

Q125. Let M = $\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ Find the unique linear transformation T : $R^3 \rightarrow R^3$ so that M is the matrix of T with respect to the basis β = {v_1 = (1, 0, 0) v_2 = (1, 1, 0) v_3 = (1, 1, 1)} of R^3 and β' = {w_1 = (1, 0), w_2 = (1, 1)} of R^2. Also find T(x, y, z)

(Year 2010)

(20 Marks)

Q126. Let A and B be nxn matrices over real's. Show that BA is invertible if I – AB is invertible. Deduce That AB and AB have same Eigen values.

(Year 2010)

(20 Marks)

Q127. In the spacer R^n. Determine whether or not the $\{e_1 - e_2, e_2 - e_3, \ldots. e_{n-1} - e_n\}$ set is linearly independent.

(Year 2010)

(10 Marks)

Q128. Let T be a linear transformation from a vector V space over real's into V such that $T - T^2 = I$. Show that is invertible.

(Year 2010)

(10 Marks)

Q129. Let A be a non-singular nxn, square matrix. Show that A.(adjA) = |A|.I_n Hence show that |adj(adjA)| = $|A|^{(n-1)^2}$

(Year 2011)

(10 Marks)

Q130. Let A = $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$ Solve the system of equations given by AX = B using the above, also solve the system of equations $A^TX = B$ where A^T denotes the transpose matrix of A.

(Year 2011)

(10 Marks)

Q131. Let $\lambda_1, \lambda_2, \ldots \lambda_n$ be the Eigen values of a nxn square matrix A with corresponding Eigen vectors $X_1, X_2, \ldots. X_n$. If B is a matrix similar to show that the Eigen values of B are same as that of A. Also find the relation between the Eigen vectors of B and Eigen vectors of A.

(Year 2011)

(10 Marks)

Q132. Show that the subspaces of R^3 spanned by two sets of vectors {(1, 1, -1), (1, 0, 1)} and {(1, 2, -3), (5, 2, 1)} are identical. Also find the dimension of this subspace.

(Year 2011)

(10 Marks)

Q133. Find the nullity and a basis of the null space of the linear transformation $A : R^4 \rightarrow R^4$ given by the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

(Year 2011)

(10 Marks)

Q134. Show that the vectors (1, 1, 1), (2, 1, 2) and (1, 2, 3) are linearly independent in R^3. Let $R^3 \rightarrow R^3$ be a linear transformation defined by T(x, y, z) = (x + 2y + 3z, x + 2y + 5z, 2x + 4y + 6z) show that the images of above under are linearly dependent. Give the reason for the same.

(Year 2011)

(10 Marks)

Q135. Let A = $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3x3. Find the Eigen values of the matrix B^3 where $B = C^{-1}AC$.

(Year 2011)

(10 Marks)

Q136. Prove or disapprove the following statement: if $B = \{b_1, b_2, b_3, b_4, b_5\}$ is a basis for R^5 and V is a two dimensional subspace of R^5, then V has a basis made of two members of B.

(Year 2012)

(12 Marks)

Q137. Let $T : R^3 \rightarrow R^3$ be the linear transformation defined by

$T(\alpha, \beta, \gamma) = (\alpha + 2\beta - 3\gamma, 2\alpha + 5\beta - 4\gamma, \alpha + 4\beta + \gamma)$. Find a basis and the dimension of the image of T and the kernel of T.

(Year 2012)

(12 Marks)

Q138. Let V be the vector space of all 2x2 matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V ? Justify your answer

(Year 2012)

(8 Marks)

Q139. Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation:

$x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$

$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0$

$3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$

(Year 2012)

(12 Marks)

Q140. Consider the mapping $f: R^2 \rightarrow R^2$ by $f(x, y) = (3x + 4y, 2x - 5y)$. Find the matrix A relative to the basis (1, 0), (0, 1) and the matrix B relative to the basis (1, 2), (2, 3)

(Year 2012)

(12 Marks)

Q141. If λ is a characteristic root of a non-singular matrix A then prove that $|A| / \lambda$ is a characteristic root of Adj.A

(Year 2012)

(8 Marks)

Q142 Let H = $\begin{bmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{bmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that D = $P^T H P'$ is diagonal.

(Year 2012)

(20 Marks)

Q143. Find the inverse of matrix A = $\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$ By using elementary row operations.

Hence solve the system of linear equations

$x + 3y + z = 10$

$2x - y + 7z = 12$

$3x + 2y - z = 4$

(Year 2013)

(10 Marks)

Q144. Let A be a square matrix and A* be its adjoint, show that the Eigen values of matrices AA* and A* A are real. Further show that trace (AA*) = Trace (A* A)

(Year 2013)

(10 Marks)

Q145. Let P_n denote the vector space of all real polynomials of degree at most n and T : $P_2 \rightarrow P_3$ be linear transformation given by $T\left(f(x)\right) = \int_0^x p(t)dt,$ p(x) ϵ P_2. Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1 + x^2, 1 + x^3\}$ of P_2 and P_3 respectively. Also find the null space of T.

(Year 2013)

(10 Marks)

Q146. Let V be an n-dimensional vector space and T : V → V be an invertible linear operator. If $\beta = \{X_1, X_2, \ldots X_n\}$is a basis of V, show that $\beta' = \{TX_1, TX_2 \ldots TX_n\}$ is also a basis of V.

(Year 2013)

(8 Marks)

Q147. Let A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where ω ($\neq 1$) is a cube root of unity. If λ_1, λ_2, λ_3, denote the Eigen values of A^2, show that $|\lambda_1| + |\lambda_2| + |\lambda_3| \leq 9$

(Year 2013)

(8 Marks)

Q148. Find the rank of the matrix A $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 5 & 8 & 12 & 17 \\ 3 & 5 & 8 & 17 & 23 \\ 8 & 12 & 17 & 23 & 30 \end{bmatrix}$

(Year 2013)

(8 Marks)

Q149. Let A be a Hermitian matrix having all distinct Eigen values λ_1, λ_2, …, λ_n. If X_1, X_2, …X_n are corresponding Eigen vectors then show that the nxn matrix C whose kth column consists of the vector X_n is non singular.

(Year 2013)

(8 Marks)

Q150. Show that the vectors $X_1 = (1, 1 + I, i)$, $X_2 = (I, -I, 1 – i)$ and $X_3 = (0, 1 – 2i, 2 – i)$ in C^3 are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers.

(Year 2013)

(8 Marks)

Q151. Using elementary row or column operations, find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(Year 2014)

(10 Marks)

Q152. Let V and W be the following subspaces of R^4 : V = {(a, b, c, d) : b – 2c + d = 0} and W = {(a, b, c, d): a = d, b = 2c}. Find a basis and the dimension of V, W, V ∩ W.

(Year 2014)

(15 Marks)

Q153. Investigate the values of λ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have

I. No solution
II. Unique solution
III. An infinite number of solutions

(Year 2014)

(10 Marks)

Q154. Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse. Also find the matrix representation $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

(Year 2014)

(10 Marks)

Q155. Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the Eigen values of A and the corresponding Eigen vectors.

(Year 2014)

(8 Marks)

Q156. Prove that Eigen values of a unitary matrix have absolute value 1.

(Year 2014)

(7 Marks)

Q157 The vectors $V_1 = (1, 1, 2, 4)$, $V_2 = (2, -1, -5, 2)$, $V_3 = (1, -1, -4, 0)$ and $V_4 = (2, 1, 1, 6)$ are linearly independent. Is it true ? Justify your answer.

(Year 2015)

(10 Marks)

Q158 Reduce the following matrix to row echelon form and hence find its rank:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$

(Year 2015)

(10 Marks)

Q159. If matrix A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then find A^{30}.

(Year 2015)

(10 Marks)

Q160. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(Year 2015)

(12 Marks)

Q161. Let $V = R^3$ and $T \epsilon A(V)$, for all $a_i \epsilon A(V)$, be defined by

$T(a_1, a_2, a_3) = (2a_1 + 5a_2 + a_3, -3a_1 + a_2 - a_3, a_1 + 2a_2 + 3a_3)$. What is the matrix T relative to the basis $V_1 = (1, 0, 1)$, $VV_2 = (-1, 2, 1)$, $V_3 = (3, -1, 1)$?

(Year 2015)

(12 Marks)

Q162. Find the dimension of the subspace of R^4, spanned by the set

{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)}. Hence find its basis.

(Year 2015)

(12 Marks)

Q163. Using elementary row operations, find the inverse of A = $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

(Year 2016)

(6 Marks)

Q164. If A = $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ then find $A^{14} + 3A - 2I$

(Year 2016)

(4 Marks)

Q165. Using elementary row operation find the condition that the linear equations have a solution:

$x - 2y + z = a$

$2x + 7y - 3z = b$

$3 + 5y - 2z = c$

(Year 2016)

(7 Marks)

Q166. If

$w_1 = \{(x, y, z) | x + y - z = 0\}$, $w_2 = \{(x, y, z) | 3x + y - 2z = 0\}$, $w_3 = \{(x, y, z) | x - 7y + 3z = 0\}$ then find $\dim(w_1 \cap w_2 \cap w_3)$ and $\dim(w_1 + w_2)$.

(Year 2016)

(3 Marks)

Q167. If $M_2(R)$ is space of real matrices of order 2x2 and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of $T: M_2(R) \to P_2(x)$ such that

$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + b + c + (a - d)x + (b + c)x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$ further find null space of T.

(Year 2016)

(10 Marks)

Q168. If T: $P_2(x) \rightarrow P_3(x)$ is such that $T(f(x)) = f(x) + T\big(f(x)\big) = f(x) + 5\int_0^x f(t)dt$, then choosing $\{1, 1 + x, 1 - x^2\}$ and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively find the matrix of T.

(Year 2016)

(6 Marks)

Q169. If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the Eigen values and Eigenvectors of A.

(Year 2016)

(6 Marks)

Q170. Prove that Eigen values of a Hermitian matrix are all real.

(Year 2016)

(8 Marks)

Q171. If A = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation T: $P_2(x) \rightarrow P_2(x)$ with respect to the bases $\{1 - x, x(1 - x), x(1 + x)\}$ and $\{1, 1 + x, 1 + x^2\}$ then find T.

(Year 2016)

(18 Marks)

Q172. Let A = $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is diagonal matrix.

(Year 2017)

(10 Marks)

Q.173. Show that similar matrices have the same characteristic polynomial.

(Year 2017)

(10 Marks)

Q174 Suppose U and W are distinct four dimensional subspaces of a vector space V, when $\dim V = 6$. Find the possible dimensions of subspace $U \cap W$.

(Year 2017)

(10 Marks)

Q175 Consider the matrix mapping $A: R^4 \to R^3$, where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Find a basis and dimension of the image of A and those of kernel A.

(Year 2017)

(15 Marks)

Q176.. Prove that the distance non-zero eigen vectors of a matrix are linearly independent.

(Year 2017)

(10 Marks)

Q177. Consider the following system of equation in x, y, z

$x + 2y + 2z = 1$

$x + ay + 3z = 3$

$x + 11y + az = b$

I. For which values of 'a' does that system have a unique ?

II. For which of values (a, b) does the system have more than one solution?

(Year 2017)
(15 Marks)

Q178. Let A be a 3x2 matrix and B a 2x3 matrix. Show that C = A.B is a singular matrix.

(Year 2018)
(10 Marks)

Q179 Show that if A and B are similar nxn matrices, then they have the same Eigen values.

(Year 2018)
(12 Marks)

Q180. For the system of linear equations

$x + 3y - 2z = -1$

$5y + 3z = -8$

$x - 2y - 5z = 7$, determine the following statements, which are true or false:

I. The system has no solution
II. The system has unique solution
III. The system has infinitely many solutions

(Year 2018)

(12 Marks)

Q181. Let $T : R^2 \rightarrow R^2$ be a linear map such that T(2, 1) = (5, 7) and T(1, 2) = (3, 3). If A is the matrix corresponding to T with respect to the standard bases e_1, e_2, then find Rank(A).

(Year 2019)
(10 Marks)

Q182. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$

The show that $AB = 6I_3$. Use this result to solve the following system of equations:

$2x + y + z = 5$

$x - y = 0$

$2x + y - z = 1$

(Year 2019)

(10 Marks)

Q183. Let A and B be two orthogonal matrices of same order and det A + det B = 0, show that A + B is a singular matrix.

(Year 2019)
(15 Marks)

Q184. Let A = $\begin{bmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{bmatrix}$

I. Find the rank of the matrix A.
II. Find the dimension of the subspace

$$V = \left\{ (x1, x2, x3, x4) \in R^4 \mid A \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = 0 \right\}$$

(Year 2019)
(15 + 5 Marks)

Q185. A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, State the Cayley-Hamilton theorem. Use this theorem to find A^{100}.

(Year 2019)
(15 Marks)

CIVIL SERVICES

PREVIOUS YEAR QUESTIONS

SEGMENT- WISE

LINEAR ALGEBRA

1. Prove that any set of n linearly independent vectors in a vector space V of dimension n constitutes a basis for V. **[2022][10]**

2. Let $\mathbf{T}: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $\mathbf{T}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\mathbf{T}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-3\\2\\8\end{pmatrix}$. Find $\mathbf{T}\begin{pmatrix}2\\4\end{pmatrix}$. **[2022][10]**

3. Find all solutions to the following system of equations by row-reduced method:
$$\mathbf{x_1 + 2x_2 - x_3 = 2}$$
$$\mathbf{2x_1 + 3x_2 + 5x_3 = 5}$$
$$\mathbf{-x_1 - 3x_2 + 8x_3 = -1}$$
[2022][15]

4. Let the set $P = \left\{ \begin{pmatrix}x\\y\\z\end{pmatrix} \middle| \begin{matrix} x - y - z = 0 \text{ and} \\ 2x - y + z = 0 \end{matrix} \right\}$ be the collection of vectors of a vector space $\mathbb{R}^3(\mathbb{R})$. Then
   - **(i)** Prove that P is a subspace of $\mathbb{R}^3$.
   - **(ii)** Find a basis and dimension of P. **[2022][10+10]**

5. Find a linear map $T: \mathbb{R}^2 \to \mathbb{R}^2$ which rotates each vector of $\mathbb{R}^2$ by an angle θ. Also, prove that for $\theta = \frac{\pi}{2}$, T has no eigenvalue in $\mathbb{R}$. **[2022][15]**

6. If $A = \begin{bmatrix}1 & -1 & 1\\2 & -1 & 0\\1 & 0 & 0\end{bmatrix}$, then show that $A^2 = A^{-1}$ (without finding A^{-1}).
[2021][10]

7. Find the matrix associated with the linear operator on $V_3(R)$ defined by $T(a, b, c) = (a + b, a - b, 2c)$ with respect to the ordered basis $B = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$.
[2021][10]

8. Show that $S = \{(x, 2y, 3x): x, y$ are real numbers$\}$ is a subspace of $R^3(R)$. Find two bases of S. Also find the dimension of S. **[2021][15]**

9. Prove that the eigen vectors, corresponding to two distinct eigen values of a real symmetric matrix, are orthogonal. **[2021][8]**

10. For two square matrices A and B of order 2, show that trace $(AB) = \textbf{trace}\ (BA)$. Hence show that $AB - BA \neq I_2$, where I_2 is an identity matrix of order 2 **[2021][7]**

11. Reduce the following matrix to a row-reduced echelon form and hence also, find its rank:

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$$ **[2021] [10]**

12. Find the eigen values and the corresponding eigen vectors of the matrix $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, over the complex-number field. **[2021][10]**

13. Consider the set V of all $n \times n$ real magic squares. Show that V is a vector space over R. Give examples of two distinct 2×2 magic squares. **[2020][10]**

14. Let $M_2(R)$ be the vector space of all 2×2 real matrices. Let $B = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}$. Suppose $T: M_2(R) \to M_2(R)$ is a linear transformation defined by $T(A) = BA$. Find the rank and nullity of T. Find a matrix A which maps to the null matrix. **[2020][10]**

15. Define an $n \times n$ matrix as $A = I - 2u \cdot u^T$, where u is a unit column vector.

(i) Examine if A is symmetric.

(ii) Examine if A is orthogonal.

(iii) Show that trace $(A) = n - 2$.

(iv) Find $A_{3\times3}$, when $u = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$. **[2020][20]**

16. Let F be a subfield of complex numbers and T a function from $F^3 \to F^3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + 3x_3, 2x_1 - x_2, -3x_1 + x_2 - x_3)$. What are the conditions on a, b, c such that (a, b, c) be in the null space of T? Find the nullity of T. **[2020][15]**

17. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$

(i) Find AB.

(ii) Find $\det(A)$ and $\det(B)$.

(iii) Solve the following system of linear equations:

$$x + 2z = 3, 2x - y + 3z = 3, 4x + y + 8z = 14$$ **[2020][15]**

18. Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$. **[2020][15]**

19. Find an extreme value of the function $u = x^2 + y^2 + z^2$, subject to the condition $2x + 3y + 5z = 30$, by using Lagrange's method of undetermined multiplier. **[2020] [20]**